SLOPE ANALYSIS FOR ELASTIC PROTON-PROTON AND PROTON-ANTIPROTON SCATTERING

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Abstract

The diffraction slope parameter is investigated for elastic proton-proton and proton-antiproton scattering based on the all available experimental data at intermediate square of momentum transfer in the main. Energy dependence of the elastic diffraction slope is approximated by various analytic functions in a model-independent fashion. The expanded standard logarithmic approximations allow to describe experimental slopes in all available energy range at qualitative level reasonably. Various fitting functions differ from each other both in low energy and very high energy domains. Predictions for diffraction slope parameter are obtained for elastic proton-proton scattering at NICA, RHIC and LHC energies, for proton-antiproton elastic reaction in FAIR energy domain for various approximation functions at intermediate square of momentum transfer. Difference of nuclear slopes for proton-antiproton and proton-proton scattering is investigated in wide momentum transfer range also.

1 Introduction

In the absence of a pure QCD description of the elastic $pp/\bar{p}p$ and these large-distance scattering states (soft diffraction), an empirical analysis based on (almost) model-independent fits to the physical quantities involved plays an important role in the extraction of novel information, that can contribute with the development of useful calculational schemes in the underlying field theory [1]. Therefore, empirical fits of energy dependences of global scattering parameters have been used as a important source of the model-independent information. This approach for σ_{tot} and ρ was recently used in [2]. The third important quantity for nucleon elastic scattering is the slope parameter. The nuclear slope B for elastic scattering defined according to the following equation with taking into account the t-dependence:

$$B\left(s,t\right)=\frac{\partial}{\partial t}\left(\ln\frac{\partial\sigma\left(s,t\right)}{\partial t}\right), \tag{1}$$

is determined experimentally. This quantity is of interest in its own right, especially for large-distance hadronic physics. On the other hand the study of B parameter is important, in particular, for reconstruction procedure of full set of helicity amplitudes for elastic nucleon scattering [2]. The present status of slope for elastic pp and $\bar{p}p$ scattering is discussed for various $|t|$ ranges over the full energy domain.

2 Slope energy dependence

We have attempted to describe the energy behaviour of the elastic nuclear slopes for pp and $\bar{p}p$ reactions. The following analytic functions are used for fitting of experimental slope energy

dependences here:

$$B(s,t) = B_0(t) + 2a_1(t)\ln(s/s_0), \quad (2a)$$
$$B(s,t) = B_0(t) + 2a_1(t)\ln(s/s_0) + a_2(t)\left[\ln(s/s_0)\right]^{a_3(t)}, \quad (2b)$$
$$B(s,t) = B_0(t) + 2a_1(t)\ln(s/s_0) + a_2(t)(s/s_0)^{a_3(t)}, \quad (2c)$$
$$B(s,t) = B_0(t) + 2a_1(t)\ln(s/s_0) + a_2(t)\left[\ln(s/s_0)\right]^2, \quad (2d)$$

where $s_0 = 1$ GeV^2. These functions were used at study of slope energy dependence in low $|t|$ domain [3].

Most of experimental investigations as well as theoretical models are focused on the diffraction region $|t| \simeq 0 - 0.5$ GeV^2. The energy dependence of experimental nuclear slope at low momentum transfer was analyzed in detail recently [3]. Specifically, we have focused on the intermediate $|t|$ domain. Experimental data are from [4 – 9]. In the intermediate $|t|$ domain experimental data set is 141 / 85 for $pp/\bar{p}p$ reaction respectively. It seems the mean value of $|t|$ is more important for separation of experimental results on different $|t|$ domains than the $|t|$-boundaries of corresponding measurements. It should be emphasized that the experimental data for intermediate $|t|$ range were separated on two samples which corresponded the various parameterization types for differential cross-section, namely, linear, $\ln(d\sigma/dt) \propto (-Bt)$, and quadratic, $\ln(d\sigma/dt) \propto (-Bt - Ct^2)$, function. As known the measurements of nuclear slope, especially at intermediate $|t|$ do not form a smooth set in energy, unlike the situation for global scattering parameters ρ and σ_{tot}, where there is a good agreement between various group data [10]. Thus the data samples for approximations are some smaller because of exception of points which differ from the other experimental points at close energies significantly. The maximum fraction of excluded points is equal 15.3% at intermediate $|t|$ values.

Figure 1 shows the experimental data and corresponding fits for slope parameter energy dependence at intermediate $|t|$ for pp and $\bar{p}p$ elastic scattering. The Fig.1a and Fig.1c correspond the linear approximation of differential cross-section for for pp and $\bar{p}p$ respectively. Experimental data obtained at quadratic fit of $d\sigma/dt$ and fitting functions (2a) – (2d) are presented at Fig.1b for pp and at Fig.1d for $\bar{p}p$ collisions. The fitting parameter values are indicated in Table 1 for various interaction types and for different $d\sigma/dt$ parameterizations. Usually the fit qualities are poorer for intermediate $|t|$ values than that for low $|t|$ range. The fitting functions (2a) and (2d) agree with experimental points qualitatively both for linear (Fig.1a) and quadratic (Fig.1b) parameterizations of $d\sigma/dt$ for $\sqrt{s} \geq 5$ GeV only. The "expanded" parameterizations (2b), (2c) approximate experimental data at all energies reasonably. But the (2c) function shows a very slow growth of slope parameter with energy increasing at $\sqrt{s} \geq 10^2$ GeV (Fig.1a). It should be stressed that the fitting function (2d) predicts decreasing of the nuclear slope in high energy domain. Such behavior is opposite the other fitting function (2a) – (2c). The $\bar{p}p$ experimental points from linear parameterization of differential cross-section are fitted by (2a) at $\sqrt{s} \geq 5$ GeV. The $\bar{p}p$ data disagreement with Regge-like fitting function very significantly (Fig.1c). One can see that the experimental data admit the approximation by (2d) for all energy range but not only for $\sqrt{s} \geq 5$ GeV. Indeed the fit quality for the first case much better than for second one. The parameter values are shown in Table 1 for approximation by (2d) of all available experimental data. The functions (2c) and (2d) show a very close behaviour at all energies for $\bar{p}p$ data from linear $d\sigma/dt$ parameterization. These fitting functions have a better fit quality than (2b). The $\bar{p}p$ data from quadratic parameterization of $d\sigma/dt$ are fitted by (2a) and (2d) functions for $\sqrt{s} \geq 5$ GeV only and for all available energies (Fig.1d). In the last case the fit qualities are much better and fitting parameters are indicated in the Table 1 for this energy range namely. As above the functions (2c) and (2d) show a very close fit quality which is some better than this parameter for (2b) fitting function. One can see the fit qualities for (2b) – (2d) are some

Table 1: Fitting parameters for slope energy dependence at intermediate $|t|$

Function	Parameter				
	B_0, GeV^{-2}	a_1, GeV^{-2}	a_2, GeV^{-2}	a_3	χ^2/n.d.f.
proton-proton scattering, experimental data for $d\sigma/dt = A\exp(-Bt)$					
(2a)	8.15 ± 0.12	0.169 ± 0.009	–	–	111/29
(2b)	10.4 ± 0.4	0.04 ± 0.01	-21.5 ± 1.5	-2.11 ± 0.12	204/55
(2c)	8.8 ± 0.2	0.13 ± 0.01	-64 ± 5	-1.39 ± 0.06	213/55
(2d)	4.06 ± 0.06	0.9 ± 0.1	-0.12 ± 0.02	–	60/28
proton-proton scattering, experimental data for $d\sigma/dt = A\exp(-Bt - Ct^2)$					
(2a)	7.1 ± 0.2	0.33 ± 0.02	–	–	193/34
(2b)	7.9 ± 0.5	0.26 ± 0.04	-10 ± 3	-3.0 ± 0.8	294/61
(2c)	7.5 ± 0.2	0.29 ± 0.02	-48 ± 30	-2.0 ± 0.4	293/61
(2d)	4.0 ± 0.9	1.0 ± 0.2	-0.14 ± 0.04	–	180/33
proton-antiproton scattering, experimental data for $d\sigma/dt = A\exp(-Bt)$					
(2a)	11.19 ± 0.05	0.138 ± 0.004	–	–	1209/27
(2b)	$(-6 \pm 2) \cdot 10^3$	0.46 ± 0.02	$(6 \pm 2) \cdot 10^3$	$(-8 \pm 3) \cdot 10^{-4}$	950/55
(2c)	-1176 ± 72	6.19 ± 0.25	1191 ± 72	$(-1.12 \pm 0.04) \cdot 10^{-2}$	719/55
(2d)	14.95 ± 0.14	-0.46 ± 0.02	0.068 ± 0.003	–	714/56
proton-antiproton scattering, experimental data for $d\sigma/dt = A\exp(-Bt - Ct^2)$					
(2a)	10.2 ± 0.2	0.189 ± 0.011	–	–	154/21
(2b)	$(-2 \pm 3) \cdot 10^3$	0.46 ± 0.05	$(2 \pm 3) \cdot 10^3$	$(-2 \pm 2) \cdot 10^{-3}$	121/19
(2c)	$(-7 \pm 2) \cdot 10^2$	4.2 ± 0.8	$(7 \pm 2) \cdot 10^2$	$(-1.4 \pm 0.2) \cdot 10^{-2}$	108/19
(2d)	14.3 ± 0.6	-0.35 ± 0.08	0.056 ± 0.008	–	108/20

better for data from quadratic parameterization of differential cross-sections than for data from linear approximation of $d\sigma/dt$. Thus the parameterizations (2b) – (2d) agree with data points at qualitative level both for linear (Fig.1c) and quadratic (Fig.1d) parameterization of proton-antiproton $d\sigma/dt$ but these fits are still statistically unacceptable.

One can get a predictions for nuclear slope parameter values for some facilities based on the results shown above. The B values at intermediate $|t|$ for different energies of FAIR, NICA, RHIC, and LHC are shown in the Table 2 based on the fitting parameters obtained for linear parameterization of $d\sigma/dt$. According to the fit range function (2a) can predicts the B value for $\bar{p}p$ scattering in $\sqrt{s} \geq 5$ GeV domain only. As expected the functions (2c) and (2d) predicted the very close slope parameter values for FAIR. All fitting functions, especially (2b) and (2c), predict the close values for nuclear slope in NICA energy domain. Functions (2a) – (2c) predict larger values for B in high-energy pp collisions than (2d) approximation. Perhaps, the future more precise RHIC results will agree better with predictions based on experimental data fits under study. The function (2d) with obtained parameters predicts negative B values at LHC energies. It should be emphasized that various phenomenological models predict a very sharp decreasing of nuclear slope in the range $|t| \sim 0.3 - 0.5$ GeV^2 at LHC energy $\sqrt{s} = 14$ TeV [11]. Just the negative B value predicted for LHC at $\sqrt{s} = 14$ TeV by (2d) is most close to the some model expectations [12, 13]. Taking into account recent predictions based on the fitting functions (2a) – (2d) for low $|t|$ [3] one can suggest that the model with hadronic amplitude corresponding to the exchange of three pomerons [13] describes the nuclear slope some closer to the experimentally inspired values at LHC energy both at low and intermediate $|t|$ than other models.

Phenomenological models predicts the zero difference of slopes (ΔB) for proton-antiproton

Table 2: Predictions for B based on the functions (2a) – (2d) for intermediate $|t|$ domain

Fitting function	Facility energies, $\sqrt{s}$									
	FAIR, GeV			NICA, GeV		RHIC, TeV		LHC, TeV		
	5	6.5	14.7	20	25	0.2	0.5	14	28	42*
(2a)	12.08	12.22	12.67	10.18	10.33	11.73	12.35	14.60	15.07	15.35
(2b)	12.46	12.23	12.03	10.39	10.49	11.10	11.29	11.88	12.00	12.07
(2c)	12.68	12.44	11.96	10.34	10.47	11.56	12.03	13.76	14.12	14.34
(2d)	12.69	12.46	11.97	10.54	10.67	9.66	7.89	-5.32	-9.41	-12.01

*The ultimate energy upgrade of LHC project [14].

and proton-proton elastic scattering at asymptotic energies. Here the difference ΔB is calculated for each function (2a) – (2d) with parameters corresponded $\bar{p}p$ and pp fits: $\Delta B_i(s) = B_i^{\bar{p}p}(s) - B_i^{pp}(s)$, $i = 2a,...2d$[1]. It should be stressed that the equal energy domain are used in $\bar{p}p$ and pp fits for ΔB calculations, i.e. the parameter values obtained by (2d) fitting function of $\bar{p}p$ data from linear fit of $d\sigma/dt$ for $\sqrt{s} \geq 5$ GeV are used for corresponding ΔB definition. The difference $\Delta B_i(s)$ at low $|t|$ values was calculated based on the recent results from [3]. The energy dependences of ΔB are shown at Fig.2a and Fig.2b for low and intermediate $|t|$ respectively. One can see that the difference of slopes decreasing with increasing of energy for low $|t|$ domain (Fig.2a). At present the proton-proton experimental data at highest available energy 200 GeV don't contradict with fast (square of logarithm of energy) increasing of slope at high energies in general case [3]. Such behavior could be agree with the asymptotic growth of total cross section. But on the other hand the quadratic function (2d) leads to very significant difference ΔB for $\bar{p}p$ and pp scattering in high energy domain for both low (Fig.2a) and intermediate (Fig.2b) values of $|t|$. The only Regge-like function (2a) predicts the decreasing of ΔB with energy growth at intermediate $|t|$ (Fig.2b). The parameterizations (2b) – (2d) predict the decreasing of difference of slopes at low and intermediate energies and fast increasing of ΔB at higher energies for intermediate $|t|$ domain (Fig.2b). As expected the most slow changing of ΔB is predicted by Regge-like parametrization (2a) at asymptotic energies. All fitting functions with experimentally inspired parameters don't predict the constant zero values of ΔB at high energies. But it should be emphasized that only separate fits were made for experimental data for pp and $\bar{p}p$ elastic reactions here. These results indicate on the importance of investigations at ultra-high energies both pp and $\bar{p}p$ elastic scattering for many fundamental questions and predictions connected to the general asymptotic properties of hadronic physics.

3 Summary

The main results of this paper are following. The most of all available experimental data for slope parameter in elastic nucleon collisions are approximated by different analytic functions. The situation is more unclear at intermediate $|t|$ values than for low $|t|$ domain. Only the qualitative agreement is observed between approximations and experimental points both for pp and $\bar{p}p$ collisions because of poorer quality of data. But the suggested "expanded" approximations can be used as a reliable fits for wide range of momentum transfer at all energies. Predictions for slope parameter are obtained for elastic proton-proton and proton-antiproton scattering in energy domains of some facilities. It seems the phenomenological model with

[1]Obviously, one can suggest various combinations of fitting functions for ΔB calculations.

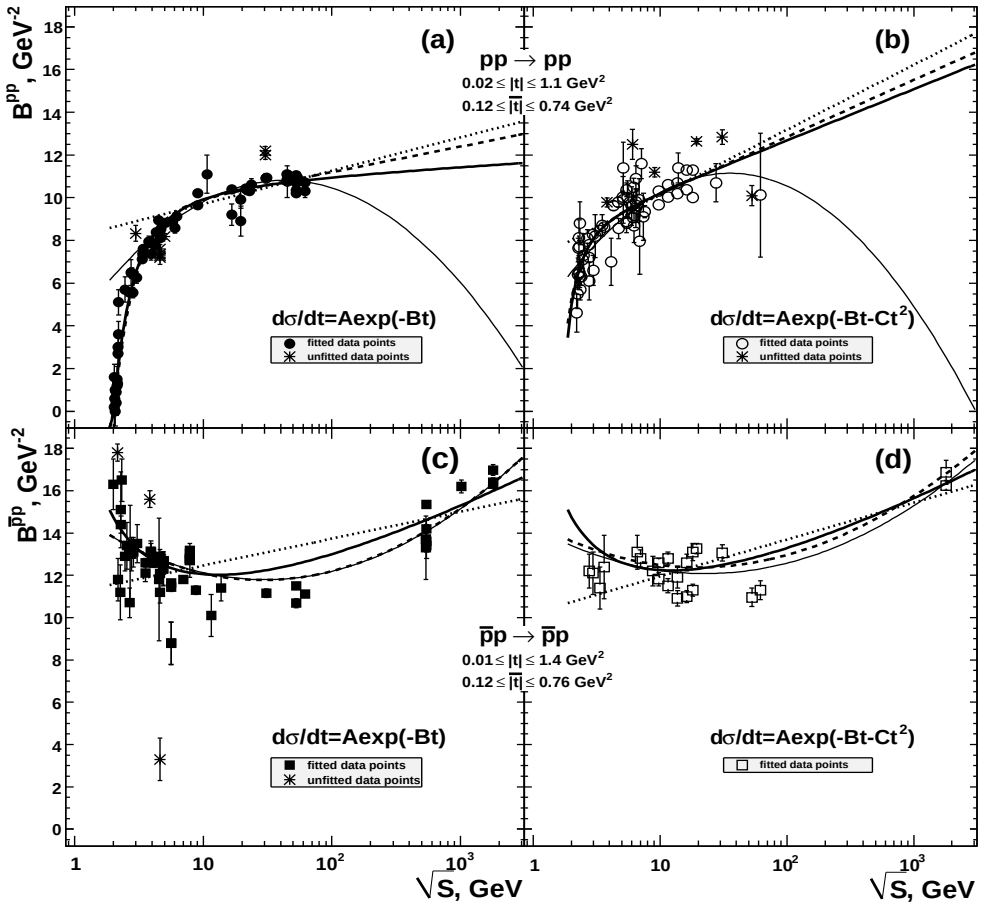

Figure 1: Energy dependence of the elastic slope parameters for proton-proton (a,b) and proton-antiproton (c,d) scattering in intermediate $|t|$ domain for linear (a,c) and quadratic (b,d) approximation of differential cross-section. The curves correspond to the fitting functions as following: (2a) – dot, (2b) – thick solid, (2c) – dot-dashed, (2d) – thin solid.

hadronic amplitude corresponding to the exchange of three pomerons describes the nuclear slope some closer to the experimental fit inspired values at LHC energy both at low and intermediate $|t|$ than other models. The energy dependence of difference of slopes (ΔB) for proton-antiproton and proton-proton elastic scattering was obtained for fitting functions under study. The ΔB parameter shows the opposite behaviours at high energies for low and intermediate $|t|$ domains (decreasing / increasing, respectively) for all fitting functions with the exception of Regge-like one. The last function predicts the slow decreasing of ΔB with energy growth. It should be emphasized that all underlying empirical fitting functions with experimentally inspired parameter values don't predict the zero difference of slopes for proton-antiproton and proton-proton elastic scattering both at low and intermediate $|t|$ for high energy domain.

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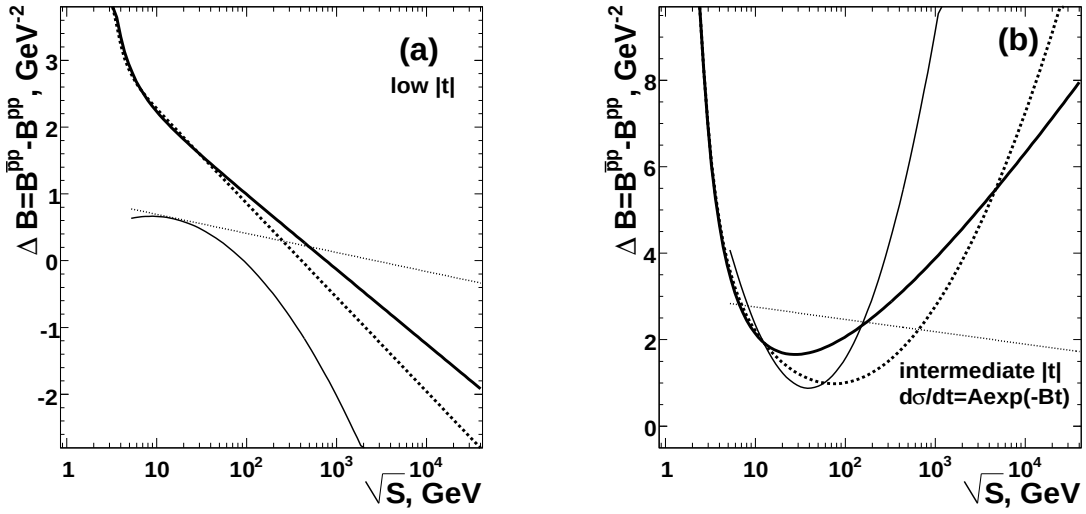

Figure 2: The energy dependence of the difference of elastic slopes for proton-antiproton and proton-proton scattering in low $|t|$ domain (a) and in intermediate $|t|$ range for linear fit of cross-section (b). The correspondence of curves to the fit functions is the same as above.

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